

Conic Sections

Answer key

HSG.GPE.A.1

HSG.GPE.A.2

15 problems

1. Identify the center and radius of the circle $(x - 3)^2 + (y + 2)^2 = 25$.

Answer Center $(3, -2)$, radius 5

Compare with $(x - h)^2 + (y - k)^2 = r^2$: $h = 3$, $k = -2$, and $r^2 = 25$ so $r = 5$.

2. Identify the center and radius of the circle $(x + 4)^2 + y^2 = 49$.

Answer Center $(-4, 0)$, radius 7

Here $h = -4$, $k = 0$, and $r^2 = 49$, so the radius is $r = 7$.

3. Write the equation of the circle with center $(-1, 5)$ and radius 3.

Answer $(x + 1)^2 + (y - 5)^2 = 9$

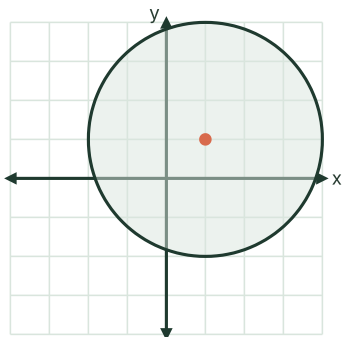
Substitute $h = -1$, $k = 5$, $r = 3$ into $(x - h)^2 + (y - k)^2 = r^2$: $(x + 1)^2 + (y - 5)^2 = 9$.

4. Write the equation of the circle whose center is the origin and which passes through the point $(6, 8)$.

Answer $x^2 + y^2 = 100$

The radius is the distance from $(0, 0)$ to $(6, 8)$: $r = \sqrt{6^2 + 8^2} = \sqrt{100} = 10$. So $x^2 + y^2 = 100$.

5. Name the center and radius of the circle graphed below.



Answer Center $(1, 1)$, radius 3

The dot is 1 right and 1 up from the origin, so the center is $(1, 1)$. The circle reaches 3 units out, so $r = 3$.

6. Convert $x^2 + y^2 - 6x + 8y + 9 = 0$ to standard form by completing the square, then state the center and radius.

Answer $(x - 3)^2 + (y + 4)^2 = 16$; center $(3, -4)$, radius 4

Group: $(x^2 - 6x) + (y^2 + 8y) = -9$. Add 9 and 16 to both sides: $(x - 3)^2 + (y + 4)^2 = -9 + 9 + 16 = 16$. Center $(3, -4)$, $r = 4$.

7. Convert $x^2 + y^2 + 10x - 4y + 13 = 0$ to standard form, then state the center and radius.

Answer $(x + 5)^2 + (y - 2)^2 = 16$; center $(-5, 2)$, radius 4

Group: $(x^2 + 10x) + (y^2 - 4y) = -13$. Add 25 and 4: $(x + 5)^2 + (y - 2)^2 = -13 + 25 + 4 = 16$. Center $(-5, 2)$, $r = 4$.

8. Which equation represents a circle with center $(2, -3)$ and radius 6?

Answer C) $(x - 2)^2 + (y + 3)^2 = 36$

Center $(2, -3)$ gives $(x - 2)^2 + (y + 3)^2$, and radius 6 means $r^2 = 36$.

9. For the parabola $y = \frac{1}{8}x^2$, state the vertex, the focus, and the directrix.

Answer Vertex $(0, 0)$, focus $(0, 2)$, directrix $y = -2$

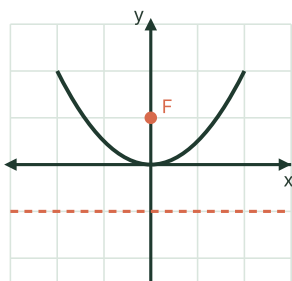
Write as $x^2 = 8y$, so $4p = 8$ and $p = 2$. The vertex is $(0, 0)$; the focus is p above it at $(0, 2)$, and the directrix is $y = -2$.

10. For the parabola $(x - 2)^2 = 12(y - 1)$, state the vertex, the focus, and the directrix.

Answer Vertex $(2, 1)$, focus $(2, 4)$, directrix $y = -2$

Here $4p = 12$, so $p = 3$, opening up. Vertex $(2, 1)$; focus is 3 above: $(2, 4)$; directrix is 3 below the vertex: $y = 1 - 3 = -2$.

11. Sketch shown below is the parabola $x^2 = 4y$. State its focus and directrix.



Answer Focus $(0, 1)$, directrix $y = -1$

From $x^2 = 4y$, $4p = 4$ so $p = 1$. The vertex is $(0, 0)$, the focus is 1 above at $(0, 1)$, and the directrix is the line $y = -1$.

12. Rewrite $y = \frac{1}{4}(x - 1)^2 - 3$ in the form $(x - h)^2 = 4p(y - k)$, then give the vertex and the value of p .

Answer $(x - 1)^2 = 4(y + 3)$; vertex $(1, -3)$, $p = 1$

Add 3: $y + 3 = \frac{1}{4}(x - 1)^2$. Multiply by 4: $(x - 1)^2 = 4(y + 3)$. So $4p = 4$, $p = 1$, and the vertex is $(1, -3)$.

13. Write the equation of the parabola with vertex $(0, 0)$ and focus $(0, 5)$.

Answer $x^2 = 20y$

The focus is 5 above the vertex, so $p = 5$ and it opens up: $x^2 = 4py = 4(5)y = 20y$.

14. A satellite dish has a cross-section shaped like the parabola $x^2 = 16y$ (units in feet), with the receiver placed at the focus. How far above the vertex of the dish should the receiver be mounted?

Answer 4 feet

From $x^2 = 16y$, $4p = 16$ so $p = 4$. The focus sits $p = 4$ feet above the vertex, so mount the receiver 4 feet up.

15. A circular garden is laid out on a coordinate grid (units in meters). A path runs along the diameter from $(2, 3)$ to $(10, 9)$. Find the center of the garden and its radius.

Answer Center $(6, 6)$, radius 5 m

Center is the midpoint: $(\frac{2+10}{2}, \frac{3+9}{2}) = (6, 6)$. Radius is half the diameter: $\frac{1}{2}\sqrt{(10-2)^2 + (9-3)^2} = \frac{1}{2}\sqrt{64+36} = \frac{1}{2}(10) = 5$.