Final Test — Form A

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28 problems

1. Describe the transformation that takes the parent $f(x) = x^2$ to $g(x) = (x-4)^2 + 3$.

Answer **Shift right 4 units and up 3 units.**

Inside, $x-4$ shifts the graph right 4; the $+3$ shifts it up 3.

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2. Simplify and write in standard form $a+bi$: $(2+5i)(3-4i)$.

Answer $26+7i$

FOIL: $6-8i+15i-20i^2 = 6+7i+20 = 26+7i$.

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3. Give the vertex of the parabola $y = -2(x+1)^2 + 5$ and state whether it opens up or down.

Answer **Vertex $(-1,5)$; opens down**

In $y = a(x-h)^2 + k$ the vertex is $(h,k) = (-1,5)$. Since $a = -2 < 0$, it opens down.

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4. Subtract and write in standard form: $(5x^3 + x^2 - 4x + 7) - (2x^3 - 3x^2 + 4x - 1)$.

Answer $3x^3 + 4x^2 - 8x + 8$

Distribute the minus: $5x^3 + x^2 - 4x + 7 - 2x^3 + 3x^2 - 4x + 1 = 3x^3 + 4x^2 - 8x + 8$.

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5. Factor completely: $27x^3 + 8$.

Answer $(3x+2)(9x^2-6x+4)$

Sum of cubes with $a = 3x,\ b = 2$: $(3x+2)\big((3x)^2 - (3x)(2) + 2^2\big) = (3x+2)(9x^2-6x+4)$.

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6. Divide using synthetic division: $(2x^3 - 5x^2 + 3x - 4) \div (x-3)$. Give the quotient and remainder.

Answer $2x^2+x+6$, **remainder** 14

Synthetic division with 3: $2,\ -5+6=1,\ 3+3=6,\ -4+18=14$. Quotient $2x^2+x+6$, remainder 14.

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7. Evaluate $16^{3/4}$, then rewrite $x^{2/5}$ in radical form.

Answer $16^{3/4} = 8$; $x^{2/5} = \sqrt[5]{x^2}$

$16^{3/4} = \left(\sqrt[4]{16}\right)^3 = 2^3 = 8$. The denominator is the index and the numerator the power: $x^{2/5} = \sqrt[5]{x^2}$.

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8. Simplify and state the restriction: $\dfrac{x^2-9x+20}{x^2-16}$.

Answer $\dfrac{x-5}{x+4},\ x \neq \pm 4$

Factor: $\dfrac{(x-4)(x-5)}{(x-4)(x+4)}$. Cancel $(x-4)$: $\dfrac{x-5}{x+4}$, with $x \neq 4$ and $x \neq -4$.

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9. Evaluate the exponential function $f(x) = 3 \cdot 2^x$ at $x = -2$.

Answer $\dfrac{3}{4}$

$f(-2) = 3 \cdot 2^{-2} = 3 \cdot \dfrac{1}{4} = \dfrac{3}{4}$.

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10. Evaluate $\log_2 32$, then expand $\log_5(25x)$ using the product property.

Answer $\log_2 32 = 5$; $\log_5(25x) = 2 + \log_5 x$

Since $2^5 = 32$, $\log_2 32 = 5$. Product rule: $\log_5(25x) = \log_5 25 + \log_5 x = 2 + \log_5 x$.

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11. Find the 15th term and the sum of the first 15 terms of the arithmetic sequence with $a_1 = 15$ and common difference $d = 8$.

Answer $a_{15} = 127$; $S_{15} = 1065$

$a_{15} = 15 + (15-1)(8) = 15 + 112 = 127$. Then $S_{15} = \frac{15}{2}(15+127) = \frac{15}{2}(142) = 1065$.

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12. Which expression is the complete factorization of $x^3 - 5x^2 + 2x - 10$?

Answer **A)** $(x-5)(x^2+2)$

Group: $x^2(x-5) + 2(x-5) = (x-5)(x^2+2)$. The factor x^2+2 does not factor over the integers.

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13. Use the quadratic formula to solve $x^2 - 6x + 13 = 0$. Write the roots in the form $a \pm bi$.

Answer $x = 3 \pm 2i$

Discriminant $= (-6)^2 - 4(1)(13) = 36 - 52 = -16$. $x = \dfrac{6 \pm \sqrt{-16}}{2} = \dfrac{6 \pm 4i}{2} = 3 \pm 2i$.

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14. Solve and check for extraneous roots: $\sqrt{2x+7} = x + 2$.

Answer $x = 1$

Square both sides: $2x + 7 = x^2 + 4x + 4$, so $x^2 + 2x - 3 = (x+3)(x-1) = 0$, giving $x = -3$ or $x = 1$. Check: $x = 1$ works ($\sqrt{9} = 3 = 1 + 2$); $x = -3$ gives $\sqrt{1} = 1 \neq -1$, extraneous. So $x = 1$.

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15. Solve and check for extraneous solutions: $\dfrac{x}{x-3} = \dfrac{3}{x-3} + 4$.

Answer **No solution (extraneous $x = 3$)**

Multiply by $(x-3)$: $x = 3 + 4(x-3) = 4x - 9$, so $9 = 3x$, $x = 3$. But $x = 3$ makes a denominator zero, so it is extraneous. There is no solution.

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16. Solve $3 \cdot 4^x = 48$.

Answer $x = 2$

Divide by 3: $4^x = 16 = 4^2$, so $x = 2$.

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17. Solve $\log_3(x-2) = 4$. Check the domain.

Answer $x = 83$

Rewrite in exponential form: $x - 2 = 3^4 = 81$, so $x = 83$. Check: $x - 2 = 81 > 0$, valid.

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18. Find the sum of the infinite geometric series $24 + 8 + \dfrac{8}{3} + \dfrac{8}{9} + \dots$

Answer $S = 36$

Here $a_1 = 24$ and $r = \dfrac{8}{24} = \dfrac{1}{3}$, with $|r| < 1$. So $S = \dfrac{24}{1 - \frac{1}{3}} = \dfrac{24}{\frac{2}{3}} = 36$.

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19. Solve the system. $\begin{cases} x^2 + y^2 = 25 \\ y = x - 1 \end{cases}$

Answer $(4,3)$ **and** $(-3,-4)$

Substitute: $x^2 + (x-1)^2 = 25 \Rightarrow 2x^2 - 2x - 24 = 0 \Rightarrow x^2 - x - 12 = (x-4)(x+3) = 0$. So $x = 4, -3$, giving $(4,3)$ and $(-3,-4)$.

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20. Solve the absolute-value inequality and write the answer as a compound statement: $|2x - 5| \leq 9$.

Answer $-2 \leq x \leq 7$

Rewrite as $-9 \leq 2x - 5 \leq 9$. Add 5: $-4 \leq 2x \leq 14$. Divide by 2: $-2 \leq x \leq 7$.

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21. A parking garage charges $\$8$ for up to 3 hours, then $\$2$ for each additional hour. The cost for t hours is $C(t) = \begin{cases} 8 & 0 < t \le 3 \\ 8 + 2(t-3) & t > 3 \end{cases}$. Find $C(2)$ and $C(7)$.

Answer $C(2) = \$8$ **and** $C(7) = \$16$

Since $2 \le 3$: $C(2) = \$8$. Since $7 > 3$: $C(7) = 8 + 2(7-3) = 8 + 8 = \$16$.

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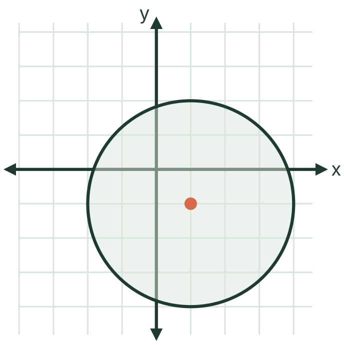
22. Name the center and radius of the circle graphed below, then write its equation.



Answer **Center** $(1, -1)$, **radius** 3; $(x-1)^2 + (y+1)^2 = 9$

The dot is 1 right and 1 down from the origin, so the center is $(1, -1)$, and the circle reaches 3 units out, so $r = 3$. Thus $(x-1)^2 + (y+1)^2 = 9$.

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23. From a club of 9 members, in how many ways can a president, vice-president, and treasurer be chosen? Then, if instead a committee of 4 (no titles) is chosen, how many committees are possible?

Answer ${}_9P_3 = 504$ **ways;** ${}_9C_4 = 126$ **committees**

Officers have distinct roles, so order matters: ${}_9P_3 = 9 \cdot 8 \cdot 7 = 504$. A committee has no order, so ${}_9C_4 = \frac{9 \cdot 8 \cdot 7 \cdot 6}{4 \cdot 3 \cdot 2 \cdot 1} = 126$.

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24. Which value is exactly equal to $\tan \frac{5\pi}{6}$?

Answer **B)** $-\frac{\sqrt{3}}{3}$

$\frac{5\pi}{6} = 150^\circ$ is in Quadrant II with reference angle 30°, where tangent is negative: $\tan \frac{5\pi}{6} = -\tan 30^\circ = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$.

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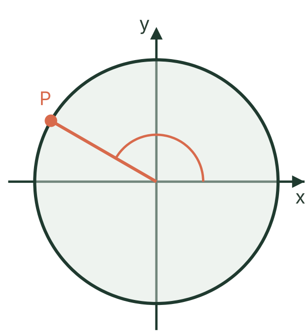
25. Give the exact coordinates $(\cos\theta, \sin\theta)$ of the point on the unit circle at $\theta = \frac{5\pi}{6}$ shown below.



Answer $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\frac{5\pi}{6} = 150^\circ$ is in Quadrant II (negative x, positive y) with reference angle 30°: $\left(\cos\frac{5\pi}{6}, \sin\frac{5\pi}{6}\right) = \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$.

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26. You deposit $\$5000$ in an account that pays 4% annual interest compounded quarterly. Using $A = P\left(1+\frac{r}{n}\right)^{nt}$ with $P = \$5000$, $r = 0.04$, $n = 4$, find the balance after 3 years, rounded to the nearest cent.

Answer $\$5634.13$

$A = 5000\left(1+\frac{0.04}{4}\right)^{4\cdot 3} = 5000(1.01)^{12} \approx 5000(1.126825) = \$5634.13.$

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27. On the first day of a savings challenge you set aside $\$1$, and each day after you set aside twice the previous day's amount ($\$1, \$2, \$4, \$8, \ldots$). What is the total amount saved after 12 days?

Answer $\$4095$

The daily amounts form a geometric series with $a_1 = 1$, $r = 2$: $S_{12} = 1 \cdot \frac{1-2^{12}}{1-2} = \frac{1-4096}{-1} = 4095$, i.e. $\$4095$.

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28. A new car is bought for $\$28{,}000$ and loses 15% of its value each year, so its value after t years is $V = 28000 \cdot (0.85)^t$. How many years until the car is first worth less than $\$14{,}000$ (half its purchase price)? Use $\frac{\ln 0.5}{\ln 0.85} \approx 4.27$.

Answer 5 **years**

Set $28000(0.85)^t = 14000$, so $(0.85)^t = 0.5$ and $t = \frac{\ln 0.5}{\ln 0.85} \approx 4.27$ years. Since value drops once per whole year, it first falls below $\$14{,}000$ after 5 years. (Check: $28000(0.85)^5 \approx \$12{,}423 < \$14{,}000$, while $28000(0.85)^4 \approx \$14{,}616$.)