NudgeLearn

Algebra 2 · Math

Logarithms

Answer key | HSF.LE.A.4 | HSF.BF.B.5

16 problems

1. Rewrite $\log_2 32 = 5$ as an exponential equation.

   Answer $2^5 = 32$

   By definition $\log_b x = y$ means $b^y = x$, so $2^5 = 32$.

2. Rewrite $3^4 = 81$ as a logarithmic equation.

   Answer $\log_3 81 = 4$

   Since $b^y = x$ means $\log_b x = y$, we get $\log_3 81 = 4$.

3. Evaluate $\log_2 64$.

   Answer 6

   Find the exponent: $2^6 = 64$, so $\log_2 64 = 6$.

4. Evaluate $\log_5 \frac{1}{25}$.

   Answer -2

   Since $5^{-2} = \frac{1}{25}$, we have $\log_5 \frac{1}{25} = -2$.

5. Evaluate $\log_8 2$.

   Answer $\frac{1}{3}$

   Since $8^{1/3} = \sqrt[3]{8} = 2$, we have $\log_8 2 = \frac{1}{3}$.

6. Evaluate $\ln e^4$.

   Answer 4

   Since $\ln x = \log_e x$, we get $\ln e^4 = 4$ (the exponent on e).

7. Expand $\log_3(9x)$ using the product property.

   Answer $2 + \log_3 x$

   $\log_3(9x) = \log_3 9 + \log_3 x = 2 + \log_3 x$, since $\log_3 9 = 2$.

8. Expand $\log\left(\frac{x^2}{y}\right)$ as a difference of logarithms.

   Answer $2\log x - \log y$

   Quotient then power rule: $\log\left(\frac{x^2}{y}\right) = \log x^2 - \log y = 2\log x - \log y$.

9. Expand $\ln\left(\frac{x^3 y}{z}\right)$ completely.

   Answer $3\ln x + \ln y - \ln z$

   Product and quotient rules give $\ln x^3 + \ln y - \ln z$, and the power rule gives $3\ln x + \ln y - \ln z$.

10. Condense $\log_2 5 + \log_2 3$ into a single logarithm.

    Answer $\log_2 15$

    By the product property, $\log_2 5 + \log_2 3 = \log_2(5 \cdot 3) = \log_2 15$.

11. Condense $3\log x - \log y$ into a single logarithm.

Answer $\log\left(\dfrac{x^3}{y}\right)$

Power rule first: $3\log x = \log x^3$. Then the quotient rule gives $\log\left(\dfrac{x^3}{y}\right)$.

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12. Condense $\frac{1}{2}\ln x + 2\ln y$ into a single logarithm.

Answer $\ln\left(\sqrt{x}\,y^2\right)$

Power rule: $\frac{1}{2}\ln x = \ln x^{1/2} = \ln\sqrt{x}$ and $2\ln y = \ln y^2$. Product rule gives $\ln\left(\sqrt{x}\,y^2\right)$.

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13. Use the change-of-base formula to write $\log_3 20$ as a quotient of common logarithms (base 10).

Answer $\dfrac{\log 20}{\log 3}$

Change of base: $\log_b x = \dfrac{\log x}{\log b}$, so $\log_3 20 = \dfrac{\log 20}{\log 3}$.

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14. Use the change-of-base formula to evaluate $\log_4 64$ as a quotient, then simplify.

Answer 3

$\log_4 64 = \dfrac{\log 64}{\log 4}$. Since $4^3 = 64$, this equals 3.

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15. Approximate $\log_2 50$ to the nearest hundredth using the change-of-base formula.

Answer ≈ 5.64

$\log_2 50 = \dfrac{\ln 50}{\ln 2} \approx \dfrac{3.9120}{0.6931} \approx 5.64$.

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16. The pH of a solution is $\mathrm{pH} = -\log[\mathrm{H}^+]$, where $[\mathrm{H}^+]$ is the hydrogen-ion concentration. Find the pH when $[\mathrm{H}^+] = 10^{-4}$.

Answer 4

$\mathrm{pH} = -\log(10^{-4}) = -(-4) = 4$.