

Quadratic Functions

Answer key

HSF.IF.C.8

HSA.REI.B.4

15 problems

1. Find the axis of symmetry of $y = x^2 - 8x + 3$.

Answer $x = 4$

The axis is $x = -\frac{b}{2a} = -\frac{-8}{2(1)} = 4$.

2. Give the vertex of the parabola written in vertex form: $y = (x - 5)^2 + 2$.

Answer $(5, 2)$

In $y = (x - h)^2 + k$ the vertex is (h, k) , so the vertex is $(5, 2)$.

3. Identify the vertex and state whether $y = -3(x + 1)^2 - 4$ opens up or down.

Answer Vertex $(-1, -4)$; opens down

Vertex form gives $(h, k) = (-1, -4)$. The leading coefficient $-3 < 0$, so it opens down.

4. Complete the square to write $y = x^2 + 4x + 1$ in vertex form.

Answer $y = (x + 2)^2 - 3$

Take half of 4 (which is 2) and square it: $y = (x^2 + 4x + 4) + 1 - 4 = (x + 2)^2 - 3$.

5. Complete the square to write $y = x^2 - 6x + 5$ in vertex form, then state the vertex.

Answer $y = (x - 3)^2 - 4$; vertex $(3, -4)$

Half of -6 is -3 , squared is 9: $y = (x^2 - 6x + 9) + 5 - 9 = (x - 3)^2 - 4$. Vertex $(3, -4)$.

6. Write $y = x^2 + 10x + 21$ in vertex form by completing the square.

Answer $y = (x + 5)^2 - 4$

Half of 10 is 5, squared is 25: $y = (x^2 + 10x + 25) + 21 - 25 = (x + 5)^2 - 4$.

7. Complete the square to write $y = 2x^2 - 12x + 13$ in vertex form.

Answer $y = 2(x - 3)^2 - 5$

Factor 2 from the x -terms: $2(x^2 - 6x) + 13 = 2(x - 3)^2 - 18 + 13 = 2(x - 3)^2 - 5$.

8. Find the minimum value of $y = x^2 - 6x + 5$ and the x -value where it occurs.

Answer Minimum -4 at $x = 3$

From vertex form $y = (x - 3)^2 - 4$, the parabola opens up, so the minimum is -4 at $x = 3$.

9. Solve for the complex roots: $x^2 + 4 = 0$.

Answer $x = \pm 2i$

$x^2 = -4$, so $x = \pm\sqrt{-4} = \pm 2i$.

10. Use the quadratic formula to solve $x^2 - 2x + 5 = 0$.

Answer $x = 1 \pm 2i$

Discriminant $= (-2)^2 - 4(1)(5) = -16$. $x = \frac{2 \pm \sqrt{-16}}{2} = \frac{2 \pm 4i}{2} = 1 \pm 2i$.

11. Use the quadratic formula to solve $x^2 + 6x + 13 = 0$.

Answer $x = -3 \pm 2i$

$$\text{Discriminant} = 6^2 - 4(1)(13) = -16. \quad x = \frac{-6 \pm 4i}{2} = -3 \pm 2i.$$

12. Which best describes the roots of $x^2 - 4x + 7 = 0$?

Answer C) Two complex (non-real) roots

$$\text{Discriminant} = (-4)^2 - 4(1)(7) = 16 - 28 = -12 < 0, \text{ so there are two complex conjugate roots.}$$

13. Use the quadratic formula to solve $2x^2 - 4x + 5 = 0$.

Answer $x = 1 \pm \frac{\sqrt{6}}{2}i$

$$\text{Discriminant} = (-4)^2 - 4(2)(5) = -24. \quad x = \frac{4 \pm \sqrt{-24}}{4} = \frac{4 \pm 2i\sqrt{6}}{4} = 1 \pm \frac{\sqrt{6}}{2}i.$$

14. A profit function is $P(x) = -x^2 + 8x - 7$, where x is the price in dollars. Complete the square to find the price that maximizes profit and the maximum profit.

Answer Vertex (4, 9): price \$4 gives a maximum profit of \$9

$$P(x) = -(x^2 - 8x) - 7 = -(x - 4)^2 + 16 - 7 = -(x - 4)^2 + 9. \text{ The vertex (4, 9) is the maximum.}$$

15. A ball is thrown upward; its height in feet is $h(t) = -16t^2 + 32t + 48$ after t seconds. Find the time it reaches maximum height and the maximum height.

Answer At $t = 1$ s, maximum height = 64 ft

$$\text{Vertex time } t = -\frac{32}{2(-16)} = 1 \text{ s. Then } h(1) = -16 + 32 + 48 = 64 \text{ ft.}$$